

**GENERALIZED DARBOUX TRANSFORM  
IN THE ISHIMORI MAGNET MODEL  
ON THE BACKGROUND OF SPIRAL STRUCTURES <sup>1</sup>**

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The integration procedure based on the the generalized Darboux transform is suggested for the Ishimori magnet model. Exact solutions are constructed for the model of background of spiral structures. The possibility of phase transition in the system is hypothesized.

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It is well known that the phenomenological approach suggested by Landau and Lifshitz in the theory of ferromagnetism is based on the idea that the evolution of weakly excited states of a spin system in the long-wavelength limit can be described in terms of a constant-length magnetization vector (magnetic moment density) and is characterized by a certain effective field [1]. With allowance for the fact that the greatest contribution to the process comes from the exchange interaction between crystals atoms, this made it possible, in particular, to obtain nonlinear equation for the one-dimensional case (isotropic Heisenberg ferromagnet model), which was subsequently solved by the inverse scattering transform method (ISM)[2, 3]: exact solitonic solutions were obtained, excitation spectrum was described, and infinite set of conservation laws was found. Further progress in this direction was achieved in work [4], where the Lax representation was applied to the anisotropic Landau-Lifshitz model (and, thereby, it was proved to belong to the class of completely integrable models), and in [5], where its exact solutions were found by the "dressing" method. Noteworthy is also work [6], which was devoted to the construction of the integrable deformations of Heisenberg model.

The situation is much more complicated in the two-dimensional case. The corresponding nonstationary equation for a Landau-Lifshitz ferromagnet, both isotropic and anisotropic, proves to be nonintegrable (see, e.g. [7]) and has exact solutions only for rather specific cases. At the same time, there is much evidence, both experimental [8] and obtained by numerical simulation [9], of the existence of stable localized two-dimensional excitations with a finite energy. For a two-dimensional system, the spectrum of these excitations becomes more diversified; in particular, new nontrivial topological objects appear in the spectrum.

The model suggested by Ishimori in [10] is presently the major tool for the phenomenological description of ferromagnets of dimensionality  $(2+1)$ . This is, primarily, due to the key property of this model: it is completely integrable and allows the use of ISM and  $\bar{\partial}$ -dressing procedure to construct a rather broad class of solutions (vortices (lumps), ra-

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tional exponential solutions, instantons, etc.) on the trivial background [11-14], while, by using a nonstandard Darboux transform [15], a physically interesting solution can be obtained in the form of a vortex circulating on circumference with a constant angular velocity.

However, it is worthy of note that the exact two-dimensional solutions were found at the expense of necessity of introducing nonlocal (along with the exchange) spin interactions into the model. The physical mechanism of this interaction is as yet unclear. In this connection, it should be emphasized that the standard Heisenberg (exchange) interaction mechanism in the so-called Schwinger-boson mean-field theory has become more understandable only a relatively short time ago (see, e.g., [8] and references cited therein). This gives grounds to hope that the nature of nonlocality that provides a broad spectrum of reasonable solutions and the corresponding observed physical objects will be clarified in the future and that the Ishimori model is quite realistic (within the framework of the adopted macroscopic approach).

In this work, a new and rather effective integration procedure is proposed for the Ishimori ferromagnet model. It opens the way for constructing exact solutions, including the ones on the nontrivial background. As to the ISM (or the  $\bar{\partial}$ -dressing method), this may encounter considerable technical difficulties.

The Ishimori magnet model is given by

$$\mathbf{S}_t = \mathbf{S} \wedge (\mathbf{S}_{xx} + \alpha^2 \mathbf{S}_{yy}) + u_y \mathbf{S}_x + u_x \mathbf{S}_y, \quad (1a)$$

$$u_{xx} - \alpha^2 u_{yy} = -2\alpha^2 \mathbf{S}(\mathbf{S}_x \wedge \mathbf{S}_y), \quad (1b)$$

where  $\mathbf{S}(x, y, t) = (S_1, S_2, S_3)$  is the three-dimensional magnetization vector,  $|\mathbf{S}| = 1$ ,  $u = u(x, y, t)$  is the auxiliary scalar real field, and the parameter  $\alpha^2$  equals  $\pm 1$ . The case  $\alpha^2 = 1$  will be called Ishimori-I magnet model (MI-I), and  $\alpha^2 = -1$  will be called MI-II.

Note that in the static limit ( $\mathbf{S} = \mathbf{S}(x, y)$ ) and  $u = \text{const}$ , the MI-I model transforms into the model of two-dimensional isotropic Heisenberg ferromagnet (elliptic version of the nonlinear  $O(3)$  - sigma-model), which was integrated in [16-17] by using the ISM with boundary conditions of the spiral-structure type.

A characteristic feature of model (1) is the presence of topological charge

$$Q_T = \frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{S}(\mathbf{S}_x \wedge \mathbf{S}_y) dx dy, \quad (2)$$

which is conserved in the course of system evolution (integral of motion) and represents the mapping of a unit 2-sphere onto the 2-sphere:  $\tilde{S}^2 \rightarrow \tilde{S}^2$ . This mapping is known to be characterized by the homotopic group  $\pi_2(\tilde{S}^2) = Z$ , where  $Z$  is the integer group, signifying that  $Q_T$  must be integer. According to Eq.(1b), the scalar function  $u = u(x, t)$  is related to the topological charge production density <sup>2</sup>.

The set of Eqs.(1) is integrated using the following associated linear system:

$$\Psi_y = \frac{1}{\alpha} S \Psi_x, \quad (3a)$$

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<sup>2</sup>Although, strictly speaking, the function  $u$  has no direct physical meaning, the functions  $u_y$  and  $u_x$ , related to each other by Eq.(1b), can likely be interpreted as "frictions coefficients" inducing forced precession of the magnetization vector  $\mathbf{S}$  along the  $x$  and  $y$  axes, respectively.

$$\Psi_t = -2iS\Psi_{xx} + Q\Psi_x, \quad (3b)$$

where  $Q = u_y I + \alpha^3 u_x S + i\alpha S_y S - iS_x$ ,  $\Psi = \Psi(x, y, t) \in \text{Mat}(2, \mathbb{C})$ ,  $S = \sum_{i=1}^3 S_i \sigma_i$ ,  $\sigma_i$  are the standard Pauli matrices, and  $I$  is a unit  $2 \times 2$  matrix. From its definition, the matrix  $S$  has the following properties:  $S = S^*$ ,  $S^2 = I$ ,  $\det S = -1$  and  $S p S = 0$  (the symbol  $(*)$  denotes Hermitian conjugation).

In what follows, we restrict ourselves to the MI-II case ( $\alpha = i$ ); the situation with MI-I can be analyzed in a similar manner.

We will solve Eq.(1) by the method of generalized Darboux matrix transform<sup>3</sup>. We also demand that system (3) be covariant ( $U \rightarrow \tilde{U}$ ,  $\Psi \rightarrow \tilde{\Psi}$ ) about the transformation of the form  $(U \equiv S)^4$

$$\tilde{\Psi} = \Omega(\Psi, \Psi_1) \Psi_1^{-1}, \quad (4)$$

where  $\Psi_1 = \Psi_1(x, y, t)$  is a certain nongenerate bare solution to the system of Eqs.(3), and  $\Omega(\Psi, \Psi_1) \equiv \Omega(x, y, t) \in \text{Mat}(2, \mathbb{C})$  is a functional defined on the pair set of matrix functions<sup>5</sup>. One then obtains from Eq.(3a) two dressing relations:

$$\tilde{U} = \Omega \Psi_1 U \Psi_1^{-1} \Omega^{-1}, \quad \tilde{U} = i\Omega_y (\Omega_x)^{-1}. \quad (5)$$

Hence, setting  $z = x + iy$ ,  $\bar{z} = x - iy$ ,  $\partial_z = (1/2)(\partial_x - i\partial_y)$ ,  $\partial_{\bar{z}} = (1/2)(\partial_x + i\partial_y)$ , and  $W^{(0)} = \Psi_1^{-1} U \Psi_1$ , we obtain the first equation for the matrix  $\Omega$  ( $\det(I \pm U) = \det(I \pm W^{(0)}) = 0$ ):

$$(I - W^{(0)})(\Omega^{-1})_{\bar{z}} = 0. \quad (6)$$

Note that, due to the symmetry relation  $\bar{U} = -\sigma_2 U \sigma_2$ , the following involutions are valid:

$$\Psi = \sigma_2 \bar{\Psi} \sigma_2, \quad Q = \sigma_2 \bar{Q} \sigma_2, \quad \Omega = \sigma_2 \bar{\Omega} \sigma_2, \quad (7)$$

which means, in particular, that Eq.(6) can be rewritten in the "conjugate" form

$$(I + W^{(0)})(\Omega^{-1})_z = 0. \quad (8)$$

To obtain the second equation for the matrix  $\Omega$ , one should use Eq.(3b). This equation, however, is rather cumbersome. Taking into account that Eqs.(5) and (6) can easily be expressed in terms of function  $\tilde{\Psi}$ , the following consideration can best be carried out for this function.

Using the identity  $Q + UQ = -2i(I + U)(uI + U)_{\bar{z}}$ , one obtain from Eq.(3b):

$$(I + U)\{\Psi_t + 2i\Psi_{xx} + 2i(u_{\bar{z}}I + U_{\bar{z}})\Psi_x\} = 0. \quad (9)$$

Let us transform this equation. Making allowance for the relation  $\Psi_{y\bar{z}} = -U_{\bar{z}}\Psi_x - iU\Psi_{x\bar{z}}$  that follows from (3b), one has

<sup>3</sup>In [18], the approach based on the Darboux transform was applied to a system that is gauge equivalent to (3), and the dressing relations were obtained thereafter.

<sup>4</sup>In the literature, the scalar variant of this transformation is sometimes called Moutard transform [19].

<sup>5</sup>It follows from Eq.(3a) that the matrix solutions  $\Psi$  and  $\Psi_1$  are related to each other by the nonlinear relationship  $\Psi_y(\Psi_x)^{-1} = \Psi_{1y}(\Psi_{1x})^{-1}$ .

$$(I + U)\{\Psi_t + 2i\Psi_{xx} + 2iu_{\bar{z}}\Psi_x - 2\Psi_{y\bar{z}} - 2iU\Psi_{x\bar{z}}\} = 0. \quad (10)$$

After multiplying both sides of this equation by  $U$  on the left and adding together the resulting expression and Eq.(10), one finds:

$$(I + U)\{\Psi_t + 2i\Psi_{zz} + 2i\Psi_{\bar{z}\bar{z}} + 4iu_{\bar{z}}\Psi_{\bar{z}}\} = 0. \quad (11)$$

Thus, after applying transformation (4) together with the covariance requirement, one obtains two (and two analogous conjugate) equations for the function  $\tilde{\Psi}$ :

$$(I - \tilde{U})\tilde{\Psi}_{\bar{z}} = 0, \quad (12)$$

$$(I + \tilde{U})\{\tilde{\Psi}_t + 2i\tilde{\Psi}_{zz} + 2i\tilde{\Psi}_{\bar{z}\bar{z}} + 4i\tilde{u}_{\bar{z}}\tilde{\Psi}_{\bar{z}}\} = 0. \quad (13)$$

Next, taking into account the identities  $\mathbf{S}(\mathbf{S}_x \wedge \mathbf{S}_y) = (1/(2i)) \text{Sp}(UU_xU_y)$  and

$$\begin{aligned} \text{Sp}(\tilde{U}\tilde{U}_x\tilde{U}_y) &= \text{Sp}(UU_xU_y) + 2i\Delta(\ln \det \tilde{\Psi}) + 2i\text{Sp}\{[U, U_x]\tilde{\Psi}^{-1}\tilde{\Psi}_x + \\ &+ [U, U_y]\tilde{\Psi}^{-1}\tilde{\Psi}_y\} + 4\text{Sp}\{U[\tilde{\Psi}^{-1}\tilde{\Psi}_y, \tilde{\Psi}^{-1}\tilde{\Psi}_x]\}, \end{aligned} \quad (14)$$

where  $\Delta$  is the two-dimensional Laplace operator, and requiring that Eq.(1b) be covariant, we rewrite it as

$$\Delta(\tilde{u} - u - 2 \ln \det \tilde{\Psi}) = 2 \text{Sp}\{[U, U_x]\tilde{\Psi}^{-1}\tilde{\Psi}_x + [U, U_y]\tilde{\Psi}^{-1}\tilde{\Psi}_y - 2iU[\tilde{\Psi}^{-1}\tilde{\Psi}_y, \tilde{\Psi}^{-1}\tilde{\Psi}_x]\}. \quad (15)$$

The relevant expression for the dressed topological charge has the form

$$\begin{aligned} \tilde{Q}_T &= Q_T + \frac{1}{4\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx dy \Delta \ln \det \tilde{\Psi} + \\ &+ \frac{1}{4\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx dy \text{Sp}\{[U, U_x]\tilde{\Psi}^{-1}\tilde{\Psi}_x + [U, U_y]\tilde{\Psi}^{-1}\tilde{\Psi}_y - 2iU[\tilde{\Psi}^{-1}\tilde{\Psi}_y, \tilde{\Psi}^{-1}\tilde{\Psi}_x]\}. \end{aligned} \quad (16)$$

Introducing the notation  $\Psi[1] = \tilde{\Psi}, \dots, U[1] = \tilde{U}, \dots, u[1] = \tilde{u}, \dots, Q_T[1] = \tilde{Q}_T, \dots$ , one can readily obtain from Eqs.(5), (15), and (16) upon  $N$ -fold dressing of the starting bare solution  $U = U^{(1)} \equiv U[0]$ <sup>6</sup>:

$$U[N] = \left( \prod_{j=0}^{N-1} \Psi[N-j] \right) U \left( \prod_{j=0}^{N-1} \Psi[N-j] \right)^{-1}, \quad (17)$$

$$u[N] = u + 2 \sum_{j=1}^N \ln \det \Psi[j] + \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx' dy' G(x - x', y - y') \times \quad (18)$$

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<sup>6</sup>These formulas can also be written in terms of the matrix functionals  $\Omega(\Psi_i, \Psi_j)$ , where  $\Psi_i, \Psi_j$  are certain bare solutions to the set of Eqs.(3) with  $S = S^{(1)}$ .

$$\begin{aligned}
& \times \sum_{j=1}^N \text{Sp } A_j(x', y', t), \\
Q_T[N] = Q_T & + \frac{1}{4\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx dy \sum_{j=1}^N \Delta \ln \det \Psi[j] + \\
& + \frac{1}{4\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx' dy' \sum_{j=1}^N \text{Sp } A_j(x', y', t),
\end{aligned} \tag{19}$$

where  $A_j(x, y, t) = \left[ U[j-1], U_x[j-1] \right] \tilde{\Psi}[j]^{-1} \tilde{\Psi}_x[j] + \left[ U[j-1], U_y[j-1] \right] \tilde{\Psi}[j]^{-1} \tilde{\Psi}_y[j] - 2iU[j-1] \left[ \tilde{\Psi}[j]^{-1} \tilde{\Psi}_y[j], \tilde{\Psi}[j]^{-1} \tilde{\Psi}_x[j] \right]$ ,  $G(x, y) = (1/2\pi) \ln(x^2 + y^2)$  is the Green's function of the Laplace equation.

Turning back to the simple dressing, we note the Eqs.(12) and (13) defines the whole collection of the solutions to model (1) (in reflectionless ( in ISM terms) section of the problem). According to Eqs.(12) and (13), four cases is possible: 1).  $\tilde{\Psi}_{\bar{z}} = 0$  and the braced expression in Eq.(13) is also zero; 2).  $\tilde{\Psi}_{\bar{z}} = 0$  and the expression in braces is nonzero; 3). the situation opposite to (2); and 4).  $\tilde{\Psi}_{\bar{z}} \neq 0$  and the expression in Eq.(13) is also nonzero (i.e., the columns of this matrix belong to the kernels of the degenerate  $I - \tilde{U} \quad I + \tilde{U}$  transformations, respectively).

In this letter, we restrict ourselves only to the first case. Then,

$$\tilde{\Psi}_{\bar{z}} = 0, \quad \tilde{\Psi}_t + 2i\tilde{\Psi}_{zz} = 0. \tag{20}$$

This system has the well-known polynomial solutions ( $\tilde{\Psi} = \{\tilde{\Psi}_{ij}\}, i, j = 1, 2, \tilde{\Psi}_{22} = \tilde{\tilde{\Psi}}_{11}, \tilde{\Psi}_{12} = -\tilde{\tilde{\Psi}}_{21}$ ) ([10], [13]):

$$\tilde{\Psi}_{11}(z, t) = \sum_{j=0}^{N_1} \sum_{m+2n=j} \frac{a_j}{m!n!} \left(-\frac{1}{2}z\right)^m \left(-\frac{1}{2}it\right)^n, \quad \tilde{\Psi}_{21}(z, t) = \sum_{j=0}^{M_1} \sum_{m+2n=j} \frac{b_j}{m!n!} \left(-\frac{1}{2}z\right)^m \left(-\frac{1}{2}it\right)^n, \tag{21}$$

where  $N_1, M_1$  are natural numbers and  $M_1 = N_1 - 1$ ,  $a_j$  and  $b_j$  are the complex numbers; and the first summation is over all possible combinations of the numbers  $m, n \geq 0$  such that  $m + 2n = j$ .

The bare solution to the set of Eqs.(1) is taken in the form of vector function  $\mathbf{S}^{(1)} = (0, \sin \Phi^{(1)}, \cos \Phi^{(1)})$ , where  $\Phi^{(1)} = \delta_0 t + \alpha_0 x + \beta_0 y + \gamma_0$ ,  $\alpha_0, \beta_0, \gamma_0, \delta_0 \in \mathbb{R}$  are parameters), i.e., in the form of a two-dimensional spiral structure with  $Q_T = Q_T^{(1)} = 0$  (according to Eq.(2)). To determine the function  $u(x, y, t) = u^{(1)}(x, y, t)$  one should substitute this vector into Eqs.(1a) and (1b). The requirement for the compatibility of the resulting two linear equations gives, after integration,

$$u^{(1)} = g_0^{(1)} \left(y + \frac{\beta_0}{\alpha_0} x\right) + \int^s g_1^{(1)}(y(s') + \frac{\beta_0}{\alpha_0} x(s'), t) ds', \tag{22}$$

where  $g_0^{(1)}$  and  $g_1^{(1)}$  are arbitrary functions. The function  $g_0^{(1)}$  is constant on the characteristic  $y + (\beta_0/\alpha_0)x = \text{const}$  and  $s$  is its parameter. Therefore, the explicit expression for  $\tilde{u}$  is determined by Eq.(18) with  $N = 1$  and by Eqs.(21) and (22).

Dressing relation (17) gives ( $S_+ = S_1 + iS_2$ )

$$\tilde{S}_3(x, y, t) = \frac{\cos \Phi^{(1)}(|\tilde{\Psi}_{11}|^2 - |\tilde{\Psi}_{21}|^2) - i \sin \Phi^{(1)}(\tilde{\Psi}_{21}\tilde{\Psi}_{11} - \tilde{\Psi}_{21}\tilde{\Psi}_{11})}{|\tilde{\Psi}_{11}|^2 + |\tilde{\Psi}_{21}|^2}, \quad (23)$$

$$\tilde{S}_+(x, y, t) = \frac{2 \cos \Phi^{(1)}\tilde{\Psi}_{21}\tilde{\Psi}_{11} + i \sin \Phi^{(1)}(\tilde{\Psi}_{11}^2 + \tilde{\Psi}_{21}^2)}{|\tilde{\Psi}_{11}|^2 + |\tilde{\Psi}_{21}|^2}.$$

Setting  $\delta_0 = 0$  and  $N_1 = 1$ , we obtain the simplest static anti(vortex) (one-lump) solution on the background of (also) static spiral structure:

$$\tilde{S}_3(x, y, t) = \frac{\cos \Phi^{(1)}[|a_0 - \frac{1}{2}a_1z|^2 - |b_0|^2] + i \sin \Phi^{(1)}[b_0(a_0 - \frac{1}{2}a_1z) - \bar{b}_0(\bar{a}_0 - \frac{1}{2}\bar{a}_1\bar{z})]}{|a_0 - \frac{1}{2}a_1z|^2 + |b_0|^2}, \quad (24)$$

$$\tilde{S}_+(x, y, t) = \frac{2b_0 \cos \Phi^{(1)}(\bar{a}_0 - \frac{1}{2}\bar{a}_1\bar{z}) + i \sin \Phi^{(1)}[b_0^2 + (\bar{a}_0 - \frac{1}{2}\bar{a}_1\bar{z})^2]}{|a_0 - \frac{1}{2}a_1z|^2 + |b_0|^2}.$$

The calculations in Eq.(16) show that  $\tilde{Q}_T \rightarrow \infty$ ; the divergence arises after the integration of the first two terms in the braces. For  $\delta_0 = \alpha_0 = \beta_0 = \gamma_0 = 0$ , solution (24) transforms into a static (anti)vortex (on the trivial background) with the topological charge  $\tilde{Q}_T = -1$  (see also [13]).

For  $\delta_0 \neq 0$  and  $N_1 = 2$ , i.e. for  $\tilde{\Psi}_{11} = a_0 - (a_1/2)z + (a_2/2)[(1/4)z^2 - it]$  and  $\tilde{\Psi}_{21} = b_0 - (1/2)b_1z$ , formulas (23) describe the dynamic (anti)two-vortex (two-lump) state on the background of (also) dynamic spiral structure with  $\tilde{Q}_T \rightarrow \infty$ , and it transforms to the state with  $\tilde{Q}_T = -2$  if the parameters entering  $\Phi^{(1)}$  turn to zero.

Clearly, the intermediate types of solutions are also quite realistic and of interest. Among these are a static vortex on the background of the dynamic spiral structure and a dynamic vortex on the background of the static spiral structure.

On the basis of these results, the hypothesis can be put forward that the structural second-order spiral-vortex  $\rightarrow$  vortex phase transition (analogous to the Kosterlitz-Thouless transition) is possible in the system of interest. This can occur if the parameters  $\delta_0$ ,  $\alpha_0$ ,  $\beta_0$  and  $\gamma_0$  are functions of time, i.e., functionals of an external nonstationary and spatially uniform magnetic field <sup>7</sup>. Then the fact that the parameter turns to zero means that there is a certain critical field ("Curie point" or, more precisely, Lifshitz point) that corresponds to the phase transition point. This hypothesis is confirmed by the experimental fact that the spiral (modulated, incommensurate) structure in a magnetic field can convert into the commensurate structure corresponding to a paramagnet with magnetic moments mainly oriented along the external field <sup>8</sup>[20]. We also emphasize that this phase

<sup>7</sup>One can show that the addition of the term  $\mathbf{S} \wedge \mathbf{H}(t)$  to the right-hand side of Eq.(1a), where  $\mathbf{H}(t)$  is the external magnetic field, merely renormalizes the magnetization vector; i.e. this term can be eliminated by the appropriate transformation through making the bare solution dependent on magnetic field.

<sup>8</sup>Clearly, the theoretical justification of this hypothesis should rest on the consideration of the order parameter of system and on the analysis of the Ginzburg-Landau functional in the vicinity of the critical point. However, although the Hamiltonian of system (1) is known (generally speaking, it is obtained in [21] for a modified MI model), the relevant calculations become overly cumbersome even for the simplest solutions and, thus, are beyond the scope of this article.

transition should be accompanied by a change in a symmetry and topological properties of the system.

Another series of solutions to model (1) can be found if the solution to the system of Eqs.(20) is sought in the form

$$\tilde{\Psi}_{11,21}(x, y, t) = \int_{-\infty}^{\infty} B_{11,21}(p) e^{-2ip^2t + pz} dp. \quad (25)$$

Here  $B_{11,21}$  are the functional parameters. In particular, by setting  $B_{11,21}(p) = c_{11,21}\delta(p - p_{11,21})$ , where  $\delta(\cdot)$  is the Dirac delta function and  $c_{11,21} \in \mathbb{C}$  and  $p_{11,21} \in \mathbb{R}$  are parameters, one gets using Eq.(17) ( $c_{11,21} \neq 0$  and the symbol c.c. stands for the complex conjugation):

$$\begin{aligned} \tilde{S}_3(x, y, t) = \\ = \frac{\cos \Phi^{(1)} [|c_{11}|^2 e^{2|p_{11}|x} - |c_{21}|^2 e^{2|p_{21}|x}] - i \sin \Phi^{(1)} [\bar{c}_{11} \bar{c}_{21} e^{2i(p_{11}^2 + p_{21}^2)t + 2(p_{11} + p_{21})\bar{z}} - c.c.]}{|c_{11}|^2 e^{2p_{11}x} + |c_{21}|^2 e^{2p_{21}x}}, \end{aligned} \quad (26)$$

$$\begin{aligned} \tilde{S}_+(x, y, t) = \\ = \frac{2\bar{c}_{11}c_{21} \cos \Phi^{(1)} e^{2i(p_{11}^2 - p_{21}^2)t + p_{11}\bar{z} + p_{21}z} + i \sin \Phi^{(1)} (\bar{c}_{11}^2 e^{4ip_{11}^2t + 2p_{11}\bar{z}} + c_{21}^2 e^{-4ip_{21}^2t + 2p_{21}z})}{|c_{11}|^2 e^{2p_{11}x} + |c_{21}|^2 e^{2p_{21}x}}. \end{aligned}$$

Therefore, an exponential and nonsingular solution is found on the background of spiral structure. At  $\Phi^{(1)} \rightarrow 0$ , the solution is finite if  $p_{11}, p_{21} > 0$ ; in this case the component  $\tilde{S}_3$  evolves only along the  $x$  variable.

More complicated solutions of this type can be found if the functionals  $B_{11,21}$  are taken as linear combinations of delta functions.

Of interest is to compare the results obtained in this work with the results of work [22], where the MI-II model was proved to be gauge equivalent to the known hydrodynamic Davey-Stuartson-II system (that describes the evolution of nearly monochromatic small-amplitude wavepacket at the surface of a small depth fluid). This means that the Lax pair can be transformed from one system to another by a certain gauge transformation, which, in turn, allows the formulas relating the solutions for these systems to be derived. It is significant that the initial boundary-value problems with specified classes should possess similar equivalence; in [22], the class of rapidly decreasing Cauchy data was assumed in both cases. Clearly, there is no gauge equivalence in the considered case of spiral (and, hence, nondecreasing) structures, while the solution constructed on their background cannot be derived from the solution to the Davey-Stuartson-II model.

Note in conclusion that the approach developed in this work can easily be extended to a series of Myrzakulov magnet models [23,24], which are modifications of the Ishimori model; for them, the first Lax-pair equation either is close or coincides with Eq. (3a) and the main modifications concern the functional  $Q$  in Eq. (3b).

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